# Pre-Calculus Summer Review Packet

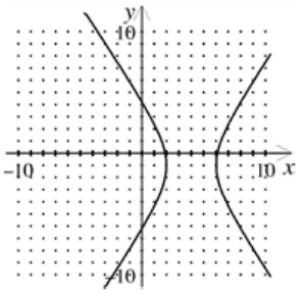
*DUE THE FIRST DAY OF SCHOOL*

The problems in this packet are designed to help you review topics from previous mathematics courses that are essential to your success in Calculus or Statistics. **You are expected to bring this completed packet to class on the first day of school**. In addition, this packet will count as part of your first quarter grade. **Upon returning, you will be ASSESSED on the content of this packet**. All contents outlined in the packet are Pre-Calculus objectives. Neatly **SHOW YOUR WORK** on a separate sheet of paper.

1. Write an equation in standard form for the ellipse with foci (7, 0) and (–7, 0) and *y*-intercepts of 6 and $-6$.

**Write the equation that represents the graph below?**

2.




**Solve:**

3. $x^2 + y^2 = 9$

   $x + y = 3$

4. The cost, *C*, of manufacturing and selling *x* units of a product is $C = 23x + 70$, and the corresponding revenue, *R*, is $R = x^2 - 70$. Find the break-even value of *x*.

5. What conic does the equation $\frac{x^2}{16} - \frac{y^2}{4} = 1$ represent?

**List the possible rational zeros of the function using the rational zeros theorem.**

6. $g(x) = x^5 - 4x^3 + 2x + 12$

**Find all zeros of the polynomial function.**

7. $f(x) = 2x^3 - 5x^2 - 2x + 2$

8. The number *C*, in thousands, of videos rented each year from a local video store can be modeled by $C = 0.096(t^3 + 3t^2 + 39t + 500)$, where $t = 0$ represents the year 1990. Using this function, what year would you expect 210.2 thousand movies to be rented?

9. Write the cubic function that passes through the points $(-1, 0)$, $(-4, 0)$, $(-6, 0)$, and $(0, 24)$?

10. Sara bought 6 fish. Every month the number of fish she has doubles. After *m* months she will have *F* fish, where $F = 6 \cdot 2^m$. How many fish will Sara have after 2 months if she keeps all of them and the fish stay healthy?

11. If $2500 is invested at a rate of 11% compounded continuously, find the balance in the account after 4 years. Use the formula $A = Pe^{rt}$.

**Solve:**

12. $\frac{1}{9} = 27^{7x-6}$

13. Solve for *x* to the nearest hundredth: $4.85^x = 17$

14. If there are initially 2000 bacteria in a culture, and the number of bacteria double each hour, the number of bacteria after *t* hours can be found using the formula $N = 2000\left(2^t\right)$. How long will it take the culture to grow to 60,000 bacteria?

15. Write an exponential function whose graph passes through the points (5, 128) and (4, 64).

16. What is the log of 100 with base 10?

17. The natural base *e* is

18. What type of function is $f(x) = 2e^{2x}$?

19. What is the solution of the equation $9^{x+1} = 27^{x-1}$?

20. What is the asymptote of the graph of $f(x) = 2^x$?

**Identify the vertical asymptote(s) of the graph of the function.**

21. $f(x) = \dfrac{4x+7}{x^2+6x+8}$

22. Graph the function $y = 1.2^x$ and its reflection over the line $y = x$. Is the reflection the graph of a function?

23. What ordered triple is the solution of the system of equations?

$$16x - 8y + 4z = -2$$
$$-8x - 4y - 8z = -8$$
$$-12x - 4y - 16z = -7$$

24. Early in the 1900s, an airplane manufacturer was able to increase the time its planes could stay aloft by constantly refining its techniques. Determine the exponential equation best models the data.

| Years after 1910 ($x$) | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Time aloft ($y$) | 0.84 | 1.3 | 2.1 | 3.6 | 4.5 | 5.9 |

**Write the expression as a complex number in standard form.**

25. $\dfrac{8+7i}{3-4i}$

26. The number of lilies a large nursery can sell each day after April 1 is modeled by a sequence whose general term is $a_n = 1700 - 75n$, where $n$ is the number of days after April 1. Find the number of lilies that can be sold on April 6th, 7th, and 8th.

**Write a rule for the *n*th term of the arithmetic sequence.**

27. –10, –4, 2, 8, . . .

28. Give the first four terms of the geometric sequence for which $a_1 = -7$ and $r = -4$.

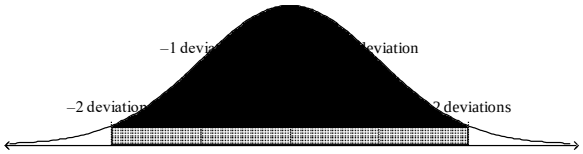
**Evaluate.**

29. $\displaystyle\sum_{j=1}^{5}\left(\frac{1}{2}\right)^j$

**Normal Distributions**

***Use the <u>Empirical Rule</u> for a normal distribution to calculate the area, percentage, relative frequency, percentile, and probability under the normal curve.***

For a normal distribution, 68.26% of the data fall between the mean and one standard deviation (34.13% of the data fall above the mean and 34.13% fall below.) Similarly, 95.48% of the data fall between the mean and two standard deviations (47.72% of the data above the mean and 47.72% below) and 99.74% of the data fall between the mean and three standard deviations above or below the mean.




30. The mean score on a normally distributed exam is 42 with a standard deviation of 12.1. What score would be expected to occur *less than* 5% of the time?

31. Which of the following is NOT TRUE about the normal distribution?

a. the mean, median and mode are equal
b. the curve is skewed to the right
c. the curve never touches the x-axis
d. the area under the curve is one.

32. Why do we use the z-score?

33. If the probability that the Islanders will beat the Rangers in a game is $\frac{2}{5}$, write the expression represents the probability that the Islanders will win *exactly* four out of seven games in a series against the Rangers?

34. $\cos(180° - \theta)$ is equal to ____.

35. Evaluate $(\csc x - \cot x)$?

36. Evaluate $(\sin 8x \cos 3x - \cos 8x \sin 3x)$?

37. Given $\sin\theta = \frac{3}{8}$, where $0 < \square < \frac{\pi}{2}$, find the value of $\sin 2\theta$. Round the answer to three decimal places.

38. Solve $8\cos x + 4 = 0$ in the interval $0° \leq x \leq 360°$. The sum of the solutions is:

39. The table shows length of time that customers had to wait in line at a sandwich shop at lunch time. The percent of customers that wait for each number of minutes is given.

| Minutes waiting | Probability |
|---|---|
| 3 | 15% |
| 4 | 15% |
| 5 | 35% |
| 6 | 25% |
| 7 | 10% |

If you get lunch at the shop for 5 days, what is the probability that you will have to spend 7 minutes waiting in line 2 of those days at most? Express your answer as a percent.

40. Suppose a survey finds that 27% of U.S. adults attend a concert at least once a month. To test this finding, you interview 50 adults at random and find that 6 of them attend a concert at least once a month. Would this result mean you should reject the survey's finding? Explain.

41. Jean's highest bowling score last season was 237 out of a possible score of 300. What percent of the maximum is Jean's high score?

42. A group is going on a boat tour. The cost, in dollars, of the tour for groups larger than 25 is given by the equation $C = 80 + 13n$, where *n* is the number of people in the group. If the cost of the tour is $600, how many are in the group?

Use for #43-47 Match the following solutions for *x* to the equations below:

a. $x = 0$
b. $x = -1$
c. $x = 4$
d. $x = 3$
e. $x = 2$
f. $x = \frac{1}{10}$

43. $2^x = 16$

44. $x = \log(1)$

45. $x^3 = 27$

46. $5^x = 25$

47. $10^x = 0.1$

48. Solve $\frac{x}{x-6} \geq -1$ by using a graph and a table.

49. Sketch the graph of $r = 2 - 4\cos\theta$.

50. Find the magnitude of the vector $W = 2\sqrt{3}\,\vec{i} + 2\vec{j}$ and the angle $\theta, 0 \leq \theta < 360$, that the vector makes with the positive *x*-axis.

51. If $U = \langle 5, 0 \rangle$ and $V = \langle 0, -4 \rangle$, find $2U - 4V$.

52. Evaluate $\csc(16.8°)$ using a calculator, round to four decimal places.

53. Convert $(-5,-4)$ to polar coordinates with $r \geq 0$ and $0° \leq \theta < 360°$. Round $\theta$ to the nearest tenth of a degree.

54. Write the complex number, $8(\cos 150° + i \sin 150°)$, in standard rectangular form.

55. Write the complex number, $1 - i$, in trigonometric form.

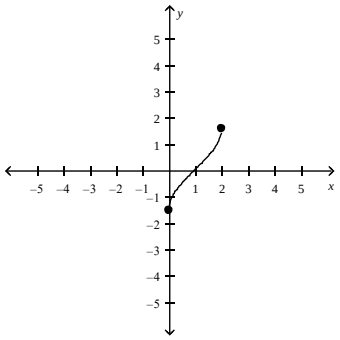
56. Use DeMoivre's Theorem to simplify the following. Write your answer in standard form.
$$(\cos 15° + i \sin 15)^{15}$$

57. DeMoivre's Theorem can be used to find reciprocals of complex numbers. Recall from algebra that the reciprocal of *x* is $\frac{1}{x}$, which can be expressed as $x^{-1}$.
Use this fact, along with DeMoivre's Theorem, to find the reciprocal of $\left(\sqrt{3} + i\right)$.

58. The horizontal and vertical components of the velocity of an arrow shot into the air are 18.0 feet per second and 29.0 feet per second, respectively. Find the velocity of the arrow. Please round your answers to the nearest tenth.

59. Evaluate $\cos\left(Tan^{-1}\left(-\frac{2}{3}\right)\right)$.

60. What function is graphed below?




61. Simplify the expression $\cos\frac{7\pi}{12} - \cos\frac{\pi}{12}$.

62. Write the equation of the circle in standard form. Identify the radius and center.
$x^2 + y^2 - 8x + 2y + 8 = 0$

63. Write the equation in standard form and classify the conic section. $5x^2 + 2y^2 + 50x - 12y + 133 = 0$

**Write an equation for the conic section.**

64. Ellipse with center (0, 0), vertex at (0, 6) and co-vertex at (5, 0)

**Classify the conic section and write its equation in standard form.**

65. $25x^2 - 4y^2 = 100$

**Solve the equation. Check for extraneous solutions.**

66. $\frac{k}{k+1} + \frac{1}{k-1} = \frac{4k-3}{(k+1)(k-1)}$

67. Evaluate the polynomial function when *y* = 5:
$f(y) = 8y^3 - 2y^2 - 3y - 5$

**List the possible rational zeros of the function using the rational zeros theorem.**

68. $h(x) = -2x^4 - 5x^3 - 3x^2 + 7x + 2$

69. The amount of money, *A*, accrued at the end of *n* years when a certain amount, *P*, is invested at a compound annual rate, *r*, is given by $A = P(1+r)^n$. If a person invests $250 in an account that pays 10% interest compounded annually, find the balance after 15 years.

70. A piece of equipment costs $75,000 new but depreciates 12% per year in each succeeding year. Find its value after 8 years.

71. Is $f(x) = 10.2e^{0.04t}$ an example of exponential growth or decay?

72. Is $f(x) = 13.7e^{-0.04t}$ an example of exponential growth or decay?

73. Condense the expression. $\frac{1}{5}\log_3 32 - 2\log_3 x + \frac{1}{2}\log_3 y$

**Solve the equation. Check for extraneous solutions.**

74. $\ln(x+7) = \ln(3x-5)$

**Use $\log 5 \approx 0.699$ and $\log 12 \approx 1.079$ to approximate the value of the expression.**

75. $\log \frac{1}{12}$

76. Tell whether (–7, –4, 1) is a solution of the following system of equations.

$$4x - 4y - 3z = -15$$
$$x - 3y - 4z = 1$$
$$5x + 3y + 4z = -37$$

**Find the inverse of the function.**

77. $y = \log_8 x$

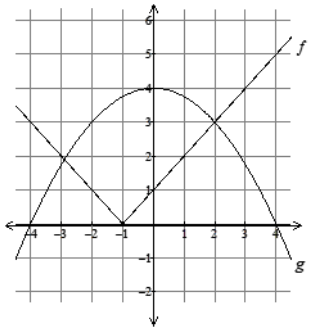
78. For the following pairs of functions, state whether they are inverses of each other.

$$f(x) = \frac{x+1}{2},\ g(x) = 2x - 1$$

**Evaluate the six trigonometric functions of $\theta$.**

79.


80. **Evaluate the following based on the graphs of $f$ and $g$ below**



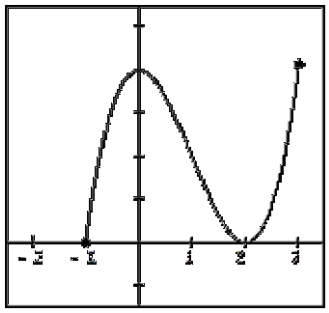

***a.*** $(f+g)(0)$

***b.*** $(fg)(-4)$

***c.*** $g \circ f(-1)$

*d.* draw a graph of $f - g$

81. Refer to the graph of *f* below:

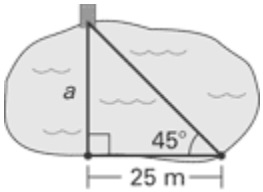


a. What is the domain of *f*?

b. Approximate the zeros of *f*.

c. Over what intervals is *f* increasing? decreasing?

82. ***Measuring Lake Width*** You want to measure the width across a lake before you swim across it. To measure the width, you plant a stake on one side of the lake, directly across from the dock. You then walk 25 meters to the right of the dock and measure a $45°$ angle between the stake and the dock. What is the width *a* of the lake?




83. The voltage *E* in an electrical circuit is given by $E = 5\cos 120\pi t$.
**a.** Find the amplitude and the period.
**b.** Find the voltage *E* when $t = 0$.

**Write the expression as a complex number in standard form.**

84. $(3 - 2i)^2$

**Find the absolute value of the complex number.**

85. $1 + 3i$

86. What is the first term of an arithmetic sequence with a common difference of 5 and a sixth term of 40?

87. Find the common difference of the arithmetic sequence.
$\frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1, \ldots$

**Normal Distributions**

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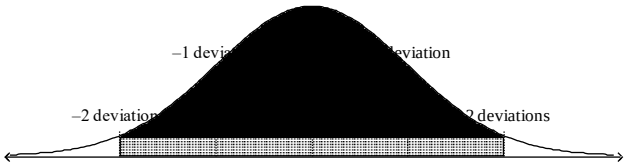


88. Professor Bartrich has 184 students in her mathematics class. The scores on the final examination are normally distributed and have a mean of 72.3 and a standard deviation of 8.9. How many students in the class can be expected to receive a score between 82 and 90?

89. In a certain school district, the ages of all new teachers hired during the last 5 years are normally distributed. Within this curve, 95.4% of the ages, centered about the mean, are between 24.6 and 37.4 years. Find the mean age and the standard deviation of the data.

90. Dr. Glendon, the school physician in charge of giving sports physicals, has compiled his information and has determined that the probability a student will be on a team is 0.39. Yesterday, Dr. Glendon examined five students chosen at random. Find, to the *nearest hundredth*, the probability that *at least* four of the five students will be on a team. Find, to the *nearest hundredth*, the probability that *exactly* one of the five students will not be on a team.

91. If $\cos \theta = \frac{4}{5}$ and $\theta$ terminates in the first quadrant, find the exact value of $\cos 2\theta$.

**Graph.**

92. $3x - 2y > -14$

93. The length of a rectangle is 7 cm more than four times the width. If the perimeter of the rectangle is 44 cm, what are its dimensions?

94. At 6:44 p.m., a parachutist is 5400 feet above the ground. At 6:52 p.m., the parachutist is 1900 feet above the ground. Find the average rate of change in feet per minute.

**Other**

95. Let $f(x) = \frac{6x+7}{2x+1}$. Find the asymptotes of the graph of *f*, and tell how the graph is related to a hyperbola with equation of the form $y = \frac{a}{x}$.

96. Open-ended Problem: Compare the quiz grades of the two algebra classes shown in the table by comparing the measures of central tendency and variation of the two data sets.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| First Period | 10 | 5 | 6 | 5 | 6 | 7 | 8 | 5 | 6 | 2 |
| Second Period | 2 | 10 | 10 | 4 | 2 | 5 | 1 | 10 | 9 | 7 |

97. A projectile is launched from ground level at an initial speed of 80 feet per second and an initial angle of $35°$.
a. Find the horizontal distance traveled by the projectile. Use the formula $x = \frac{1}{32}v^2 \sin 2\theta$ where *x* is the horizontal distance (in feet), *v* is the initial speed in feet per second, and □ is the initial angle.
b. At what angle should the projectile be launched to maximize distance? Explain.

98. Your algebra class is asked to solve the trigonometric equation $3\sin x = 1 + \sin x$ in the interval $0 \le x < 2\pi$.
a. Rewrite the equation to isolate $\sin x$ on one side of the equation.
b. Carlos solves the equation and finds the solution $x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Is he correct? Explain your answer.

99. A cubic polynomial function $f(x)$ has constant term 6 and leading coefficient 3. Suppose $f(1) = 7$ and $f(2) = 34$.

a. *Describe* how to find the coefficients of the cubic polynomial.
b. Find the cubic polynomial function $f(x)$.
c. Would changing the value of $f(1)$ or $f(2)$ change the end behavior of the function? *Explain*.

100. Verify the identity:

$$\sin(A + B) + \sin(A - B) = 2\sin A \cos B$$