ACCELERATED MATH IV SUMMER PACKET

DUE THE FIRST DAY OF SCHOOL

The problems in this packet are designed to help you review topics from previous mathematics courses that are essential to your success in Integrated Pre-Calculus Honors. **You are expected to bring this completed packet to class on the first day of school.** In addition, this packet will count as part of your first quarter grade. **Upon returning, you will be ASSESSED on the content of this packet**. All contents outlined in the packet are Integrated Math IV objectives. Neatly **SHOW YOUR WORK** on a separate sheet of paper.

1. What is the effect on the graph of the equation $y = x^2 + 1$ when it is changed to $y = x^2 - 2$?

Write in standard form and graph.

2. $y = 3(x-5)(x-6)$

Factor the expression.

3. $x^2 + 7x + 12$

4. Graph the function: $f(x) = 3^x$

5. Find the *y*-intercept of the equation. $y = -3 \cdot 7^x$

6. Solve. $e^{-0.07t} = 8$

7. What is the solution of the equation $9^{x+1} = 27^{x-1}$?

Choose the statement below that is true about the given numbers.

8.

	Column A	Column B
When $x = 0$	$y = \left(\frac{2}{3}\right)^x$	$y = \left(\frac{2}{3}\right)^{-x}$

a. The number in column A is greater.
b. The number in column B is greater.
c. The two numbers are equal.
d. The relationship cannot be determined from the given information.

Solve.

9. $3(x-8)^2 - 29 = 37$

Find the sum or difference.

10. $\left(5h^3 + 8h - 9\right) - \left(6h^3 + 6h - 4\right)$

Find the product.

11. $(u+4)\left(u^2 - 3u + 3\right)$

Divide.

12. $\left(-11x^2 - 6x^3 + 10 + 3x\right) \div (3x+1)$

13. Find the solution(s) of the equation $\sqrt[3]{x-5} = -6$

14. The wattage rating *W* (in watts) of an appliance varies jointly with the square of the current *I* (in amperes) and the resistance *R* (in ohms). If the wattage is 6 watts when the current is 0.2 ampere and the resistance is 150 ohms, find the wattage when the current is 0.3 ampere and the resistance is 300 ohms.

Simplify the rational expression, if possible.

15. $\frac{n^2 + 2n - 24}{n^2 - 11n + 28}$

Perform the indicated operation(s) and simplify.

16. $\frac{6}{x+4} + \frac{5}{x-4}$

Solve the equation. Check for extraneous solutions.

17. $\frac{x+2}{4x} - \frac{3}{2x} = \frac{1}{8}$

18. A board of length $\frac{5}{x+2}$ cm was cut into two pieces. If one piece is $\frac{7}{x-2}$ cm, express the length of the other board as a rational expression.

19. Factor $125x^3 - 64y^3$.

20. Bandar throws rocks into a quarry lake from the top of a 53 foot high wall. The chart gives the horizontal distance, *x* (in feet), the rock has traveled from Bandar and the height, *y* (in feet), of the rock above the lake.

distance, *x*	6	16	33	47
height, *y*	63.62	75.15	77.12	62.04

Determine the equation that best fits the path of the rock from Bandar to the lake

Solve the linear system.

21. $3x + y = 17$
$-4x - y = -21$

Write the expression as a complex number in standard form.

22. $\frac{8+7i}{3-4i}$

Determine which value is a solution of the equation.

23. $10 - 5w = 25$

24. The literature club is printing a storybook to raise money. The print shop charges $3 for each book, and $45 to create the film. How many books can the club print if their budget is $525?

25. Write the equation that matches the input-output table?

Input, *x*	1	2	3	4	5
Output, *y*	7	11	15	19	23

Solve.

26. $9x - 5 \le 7x - 11$

27. How would you translate the graph of $y = -x^2$ to produce the graph of $y = -(x-6)^2$?

Divide the expressions. Simplify the result.

28. $\frac{x^2 - 36}{x+5} \div (x+6)$

29. Write the standard form of the equation of the parabola with its vertex at (0, 0) and focus at $(0, 5)$.

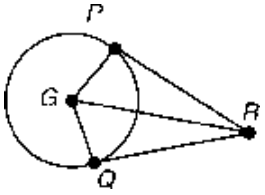
Solve:

30. $x^2 + y^2 = 9$
$x + y = 3$

31. $x^2 + y^2 = 9$
$x + y = 3$

32. The cost, *C*, of manufacturing and selling *x* units of a product is $C = 23x + 70$, and the corresponding revenue, *R*, is $R = x^2 - 70$. Find the break-even value of *x*.

33. $\overline{RP}$ is tangent to $\odot G$ at *P*. $\overline{RQ}$ is tangent to $\odot G$ at *Q*. Choose the statement that is NOT true.



a. $\angle GRP \cong \angle GRQ$
b. $\angle PGR \cong \angle QGR$
c. $\overline{PR} \cong \overline{QR}$
d. $\angle GPR$ is obtuse.

34. Sara bought 6 fish. Every month the number of fish she has doubles. After *m* months she will have *F* fish, where $F = 6 \cdot 2^m$. How many fish will Sara have after 2 months if she keeps all of them and the fish stay healthy?

35. Solve triangle *ABC* given that $A = 45°$, $B = 54°$, and $b = 70$.

36. Three ships are at sea: the *Admiral*, the *Barstow*, and the *Cauldrew*. The crew on the *Admiral* can see both the *Barstow* and the *Cauldrew*. They measure the angle between the line of sight to the *Barstow* and the line of sight to the *Cauldrew* as 31°. They radio the *Barstow* and by comparing known landmarks, find that the distance between the *Admiral* and the *Barstow* is 402 meters. The *Barstow* reports that an angle of 70° is found between their line of sight to the *Admiral* and their line of sight to the *Cauldrew*. To the nearest meter, what is the distance between the *Barstow* and the *Cauldrew*?

37. Given triangle *ABC* with $b = 2$, $c = 4$, and $m\angle A = 118°$, find *a*. Round the answer to two decimal places.

38. Graph $f(x) = -3 + e^x$.

39. Which function rule matches the input-output table?

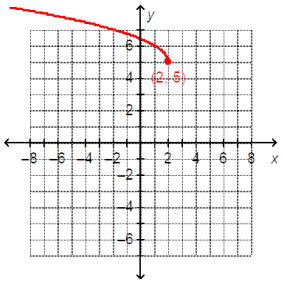
Input, *x*	1	2	3	4	5
Output, *y*	7	11	15	19	23

a. $y = 3 + 5x$ b. $y = 3 + 4x$ c. $y = 4 + 3x$ d. $y = 2 + 4x$

40. Solve triangle *ABC* given that $A = 45°$, $B = 54°$, and $b = 70$.

41. Graph the solution to the inequality $\sqrt{x+2} > 2$?

42. Write the domain of the function graphed below in interval notation.



Complete the statement to describe the end behavior of the graph of the function.

43. $f(x) = -x^3 + 7x + 4$

$f(x) \to$ _____ as $x \to -\infty$ and $f(x) \to$ _____ as $x \to +\infty$.

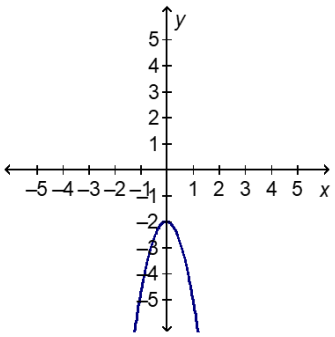
44. $f(x) = 3x^3 - x^2 + 4$

$f(x) \to$ _____ as $x \to -\infty$ and $f(x) \to$ _____ as $x \to +\infty$.

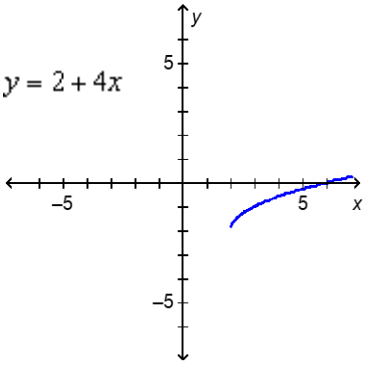
45. Use Pascal's triangle and the binomial expansion theorem to expand $(2x - 3y)^4$. What is the coefficient of the term containing xy^3?

46. Solve $(x-5)^2 = 1$.

47. What would happen to the graph if the value of *a* in $y = ax^2 + bx + c$ changed from –3 to 3?



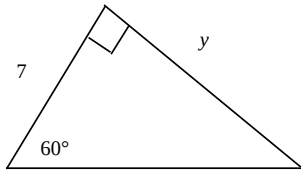
48. What is the domain and range of $y = \sqrt{x-2} - 2$?



49. A park maintenance person stands 15 m from a circular monument. If you draw two tangents from the maintenance person to each side of the monument, they make an angle of 37°. What is the measure of the arc created where the lines intersect the monument?

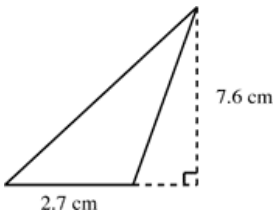
50. Rewrite as a logarithm and solve for *x* to the nearest thousandth. $5^x = 97$

51. Find the exact value of each variable.



52. A bag contains hair ribbons for a spirit rally. The bag contains 5 black ribbons and 7 green ribbons. Lila selects a ribbon at random, then Jessica selects a ribbon at random from the remaining ribbons. Find the probability that both events *A* and *B* occur. Express your answer as a fraction in simplest form.
Event *A*: Lila selects a black ribbon.
Event *B*: Jessica selects a green ribbon.

53. Find the area of the triangle.



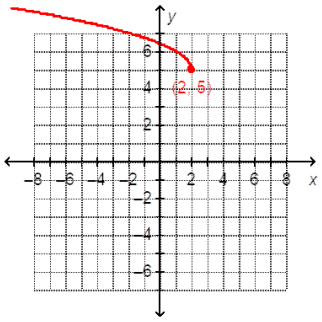
54. Rewrite in exponential form and solve for *x*.
$\ln x = 2.39$ Two students tried this problem and got different results. Erica got 245.47, but Jorie got 10.91. Who solved it correctly? Which answer is correct?

55. Express the solution to $5(e)^{24t} = 50$ as a logarithm.

56. Graph the solution to the inequality $\sqrt{x+2} > 2$?

57. Graph the function $f(x) = \frac{1}{2}x - 4$ for the domain $x \geq -1$. Classify the function as *discrete* or *continuous*. Then identify the range of the function.

58. Write the domain of the function graphed below in interval notation.



59. Does the parabola open *up* or *down*? $y = 4 + 6x - 2x^2$

60. Given: $f(x) = \sqrt{x-8}$ and $g(x) = x^2 + 9$, find $(f \circ g)x$.

61. Graph the function $f(x) = -x^2 + 2x + 8$. Show intercepts, maxima, and minima.

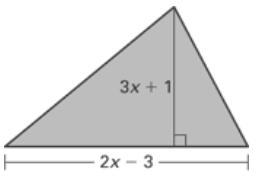
Find all real zeros of the function.

62. $p(x) = 2x^3 - 42x + 40$

63. Describe the end behavior of the function $f(x) = 3x^3 + 4x^4 - 2x^2 - 5$.

64. Simplify the expression $\frac{x^2 + 4x}{x^2 - 16}$.

65. Find an expression for the area of the figure.



Let $f(x) = 1 - x^2$ and $g(x) = \frac{1}{x}$. Find the indicated value.

66. $g(f(-5))$

67. Let $f(x) = 6x$. Find f^{-1}.

68. Are the functions *f* and *g* below inverses of each other?

$g(x) = \frac{1}{3}x - \frac{1}{2}$, $f(x) = \frac{3}{2}(2x + 1)$

69. In this exercise, refer to the division problem $\left(4x^4 - 20x^3 + 23x^2 + 5x - 6\right) \div (x - 3)$.

a. Find the quotient using long division.

b. Find the quotient using synthetic division.

c. Explain why you subtract in the process of long division, but add when using synthetic division.

d. Under what conditions can you use synthetic division to determine the quotient? Give an example where synthetic division would not be a good option.

e. What is the remainder for this problem? What information does this provide about $(x - 3)$?

f. List the possible rational zeros of the equation $f(x) = 4x^4 - 20x^3 + 23x^2 + 5x - 6$.

g. ***Critical Thinking*** Use long division to divide $(2x - 3)$ into $4x^4 - x^2 - 2x + 1$. Then use synthetic division. What do you notice about your solutions? Explain why it is still possible to use synthetic division if the leading coefficient of the linear expression is not 1. Clearly indicate why one must be careful when using this method.

70. The Modern Grocery has cashews that sell for $4.75 a pound and peanuts that sell for $2.50 a pound. How much of each must Albert, the grocer, mix to get 90 pounds of mixture that he can sell for $3.00 per pound. Express the problem as a system of linear equations and solve using the method of your choice to find the solution of the problem.

71. Solve for *v* in the equation $t = \frac{u + v}{v}$.

72. The cost of a 5 pound bag of dog food ranges from $5.25 to $5.95. Write and graph an inequality to describe this statement.

Tell whether the function is linear. Then evaluate the function when $x = -6$.

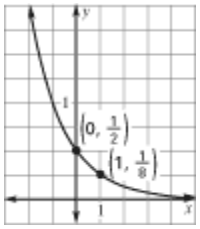
73. $f(x) = x^3 - 7$

Find the inverse of the function.

74. $f(x) = \frac{3}{4}x^3 - 1$

75. The projected worth (in millions of dollars) of a large company is modeled by the equation $y = 256(1.04)^x$. The variable *x* represents the number of years since 1997. What is the projected annual percent of growth, and what should the company be worth be in 2007?

Tell whether the graph represents *exponential growth* or *exponential decay* . Then write a rule for the function.

76. 

77. Points *Q*, *R*, and *S* lie on circle *P*. If $m\angle QPR = 90°$ and $m\angle QPS = 120°$, what are the possible measures for $\overset{\frown}{RS}$? Draw a diagram to support your explanation.

78. The volume of a cone is $256\pi \text{ in}^3$ and the height is 12 inches. Find the radius.

79. A company has a spherical storage tank which is in need of painting. The radius of the tank is 35.4 ft. The type of paint used will cover approximately 160 ft^2 per gallon. How many gallons of paint will be needed? (Round decimal to the higher whole number of gallons.)

80. A satellite is made of a cylinder and two hemispheres. The hemispheres have the same radius as the cylinder and each fit snugly on either end of the cylinder. If the diameter of the cylinder is 9 m and its length is 19 m, find the volume of the satellite. Leave your answer in terms of π.

81. A drawer contains 4 single red socks, 9 single white socks, and 2 single blue socks. Without looking, you draw out a sock, you replace it and draw out a second sock. What is the probability that the first sock and the second sock are both red?

82. Verify that $f(x)$ and $g(x)$ are inverses by showing that $(f \circ g)x = x$ and $(g \circ f)x = x$. $f(x) = 5x + 8$ and $g(x) = \frac{1}{5}(x - 8)$.

83. A piece of fabric measures 39 inches by 42 inches. A triangular scarf with a height of 21 inches and a base of 22 inches is cut from the fabric. How much is left over?

84. Graph the function $f(x) = -x^2 + 2x + 8$. Show intercepts, maxima, and minima.

85. Solve for *v* in the equation $t = \frac{u + v}{v}$.

86. The volume of a cone is $256\pi \text{in}^3$ and the height is 12 inches. Find the radius.

87. Describe how to obtain the graph of $y = \sqrt{x + 2} + 4$ from the graph of $y = \sqrt{x}$.

88. The function $C = \frac{5}{9}(F - 32)$ converts degrees Fahrenheit *F* to degrees Celsius *C*. The function $K = C + 273$ converts degrees Celsius *C* to degrees Kelvin *K*. Find the composition of the function *K* with the function *C* and explain what it represents.

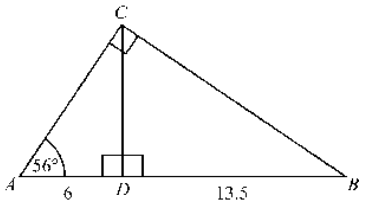
89. Open-ended: Choose any complex number $a + bi$, with $a \neq 0$ and $b \neq 0$. Show that the product of your complex number and its complex conjugate is a real number.

90. A farmer wants to fence off a portion of a square field for a vegetable garden. The length of the garden will be 4 feet less than the length of the square field. The width of the garden will be 8 feet less than the length of the square field.

    a. Using *x* as the length of the square field, write an expression for the area of the garden.
    b. If the area of the garden will be 192 square feet, what are the dimensions of the vegetable garden?
    c. Write the function $f(x) = (x - 4)(x - 8) - 192$ in standard form and explain how to use the graph of $f(x)$ to check your answer in part (b).

91. Open-ended Problem: A plumber charges $20 an hour. For each job he will charge an additional $25 for the service call. Write an equation that represents the plumber's total charge with respect to the number of hours worked. Find the inverse of your equation. What does the inverse equation represent?

92. Drawing the altitude $\overline{CD}$ from the right angle to the hypotenuse in right triangle *ABC* creates two new right triangles, ΔACD and ΔCBD, which are similar to each other and also to ΔABC. (The figure may not be drawn to scale.)



Part A Find the measures of $\angle ACD$, $\angle BCD$, and $\angle CBD$.
Part B Which angles and sides in ΔACD and ΔCBD correspond to each other? Explain your answer.
Part C Write a proportion that can be solved to find CD. Solve it and show your work.
Part D What is the length AC? Show the equation used and the steps used to solve it to find *AC* to the nearest tenth of a unit.

93. Find positive numbers *M*, *N*, and *b* (with $b \neq 1$) to show that, in general,

$\log_b \frac{M}{N} \neq \log_b M \div \log_b N$.

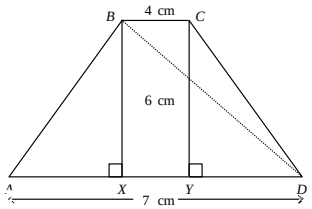
94. Use the equation $y = \frac{x^2 - 4x - 21}{x + 3}$.

a. What is the domain of the equation? Explain.

b. Graph $y = \frac{x^2 - 4x - 21}{x + 3}$. Use an open circle for a point of discontinuity, a point where the function is undefined.

c. Compare your graph from part (b) with the graph of $y = x - 7$.

95. Use the law of cosines to investigate the lengths of the diagonals of the isosceles trapezoid with bases 4 centimeters and 7 centimeters long and a height of 6 centimeters. Find *XY*, *AX*, *AB*, and cos *A*. Use this information to find *BD*.



96. A roofer says that he will only walk on a roof that has a vertical rise of 42 cm per 100 cm run. What angle does the roof make with the attic floor?

Part A Draw a diagram to model the situation.

Part B Write an appropriate trigonometric equation to solve the problem.

Part C Use technology to solve the equation to the nearest tenth.

97. At a class reunion of 40 alumni, 11 of the alumni had graduated with honors and 13 had been varsity athletes. Three of those ex-varsity athletes are honor graduates as well.

Part A Draw a Venn diagram to display the given information about numbers of alumni.

Part B What is the probability that a randomly chosen alumnus from this group had neither graduated with honors nor been a varsity athlete? Explain. Give your answer as a decimal. Round to the nearest hundredth if necessary.

Part C Suppose an alumnus is randomly chosen from those who did not graduate with honors. What is the probability that this alumnus had been a varsity athlete? Give your answer as a decimal. Round to the nearest hundredth if necessary.

Part D Suppose that two alumni are randomly chosen. Show how to find the probability that the first one chosen graduated with honors and the second one chosen neither graduated with honors nor served as a varsity athlete. Next show how to find the probability that the first one chosen neither graduated with honors nor served as a varsity athlete and the second one chosen graduated with honors. Then show how to use those two probabilities to determine the probability that a randomly chosen set of two alumni consists, in either order, of one alumnus who graduated with honors and one alumnus who neither graduated with honors nor served as a varsity athlete. Express the final answer as a decimal. Round to the nearest hundredth if necessary.

98. Use the equation $y = \frac{x^2 - 4x - 21}{x + 3}$.

a. What is the domain of the equation? Explain.

b. Graph $y = \frac{x^2 - 4x - 21}{x + 3}$. Use an open circle for a point of discontinuity, a point where the function is undefined.

c. Compare your graph from part (b) with the graph of $y = x - 7$.