

## INTEGRATED MATH IV SUMMER PACKET

### DUE THE FIRST DAY OF SCHOOL

The problems in this packet are designed to help you review topics from previous mathematics courses that are essential to your success in General Math. **You are expected to bring this completed packet to class on the first day of school.** In addition, this packet will count as part of your first quarter grade. **Upon returning, you will be ASSESSED on the content of this packet.** All contents outlined in the packet are Integrated Math IV objectives. Neatly **SHOW YOUR WORK** on a separate sheet of paper.

#### Solve the linear system.

- $3x + y = 17$   
 $-4x - y = -21$
- Ace Rent a Car charges a flat fee of \$15 and \$0.24 a mile for their cars. Acme Rent a Car charges a flat fee of \$29 and \$0.14 a mile for their cars. Use the following model to find out after how many miles Ace Rent a Car becomes more expensive than Acme Rent a Car.  
 $c = 15 + 0.24m$  Ace  
 $c = 29 + 0.14m$  Acme

#### Factor the expression.

- $64y^2 - 49$
- Solve using factoring:  $3x^2 + 5x - 12 = 0$

#### Simplify the expression.

- $2\sqrt{605} \cdot \sqrt{55}$

#### Simplify the expression.

- $(-3c^3d^4e^6)^2$

#### Find the sum or difference.

- $(5k^3 + 8k - 9) - (6k^3 + 6k - 4)$
- A rectangle has a length of  $x - 6$  and a width of  $x - 7$ . Write an equation that describes the perimeter,  $P$ , of the rectangle in terms of  $x$ .

#### Factor the polynomial completely.

- $5x^3 + 5x^2 + 30x$
- James is purchasing wood for his boss at Robertson's Diner. His boss, Cheyenne, told him that they need a piece of wood longer than 10.8 feet. What length of wood will be long enough?

- Which of the following statements are true?

- Sample size is never bigger than population size.
- The population mean is a statistic.

- Let  $f(x) = x^2 - 4$  and  $g(x) = -3x^2$ . Find  $f(g(x))$ .
  - $-3x^4 + 12$
  - $-3x^4 + 24x^2 - 48$
  - $9x^4 - 4$
  - $-3x^4 - 4$

- The literature club is printing a storybook to raise money. The print shop charges \$3 for each book, and \$45 to create the film. How many books can the club print if their budget is \$525?

#### Solve.

- $9x - 5 \leq 7x - 11$

#### Solve the absolute value inequality.

- $|m - 2| > 1$
- Sara bought 6 fish. Every month the number of fish she has doubles. After  $m$  months she will have  $F$  fish, where  $F = 6 \cdot 2^m$ . How many fish will Sara have after 2 months if she keeps all of them and the fish stay healthy?
  - 20
  - 10
  - 144
  - 24

- Graph  $f(x) = -3 + e^x$ .

- Write the standard form of the equation of the parabola with its vertex at  $(0, 0)$  and focus at  $(0, 5)$ .
- The cost,  $C$ , of manufacturing and selling  $x$  units of a product is  $C = 23x + 70$ , and the corresponding revenue,  $R$ , is  $R = x^2 - 70$ . Find the break-even value of  $x$ .
- Solve triangle  $ABC$  given that  $A = 45^\circ$ ,  $B = 54^\circ$ , and  $b = 70$ .

**Write in standard form and graph.**

21.  $y = 3(x - 5)(x - 6)$

22. Find the  $y$ -intercept of the equation.  $y = -3 \cdot 7^x$

23. The amount of money,  $A$ , accrued at the end of  $n$  years when a certain amount,  $P$ , is invested at a compound annual rate,  $r$ , is given by

$A = P(1 + r)^n$ . If a person invests \$310 in an account that pays 8% interest compounded annually, find the balance after 5 years.

24. What is the solution of the equation  $9^{x+1} = 27^{x-1}$ ?

**Solve.**

25.  $3(x - 8)^2 - 29 = 37$

**Divide.**

26.  $(-11x^2 - 6x^3 + 10 + 3x) \div (3x + 1)$

27. Find the solution(s) of the equation  $\sqrt[3]{x-5} = -6$ ?

28. The wattage rating  $W$  (in watts) of an appliance varies jointly with the square of the current  $I$  (in amperes) and the resistance  $R$  (in ohms). If the wattage is 6 watts when the current is 0.2 ampere and the resistance is 150 ohms, find the wattage when the current is 0.3 ampere and the resistance is 300 ohms.

**Solve the equation. Check for extraneous solutions.**

29.  $\frac{x+2}{4x} - \frac{3}{2x} = \frac{1}{8}$

30. A board of length  $\frac{5}{x+2}$  cm was cut into two pieces. If one piece is  $\frac{7}{x-2}$  cm, express the length of the other board as a rational expression.

**Write the expression as a complex number in standard form.**

31.  $\frac{8+7i}{3-4i}$

32. Write the equation that matches the input-output table?

Input, $x$	1	2	3	4	5
Output, $y$	7	11	15	19	23

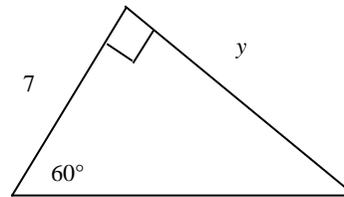
33. Write the standard form of the equation of the parabola with its vertex at  $(0, 0)$  and focus at  $(0, 5)$ .

34. Three ships are at sea: the *Admiral*, the *Barstow*, and the *Cauldrew*. The crew on the *Admiral* can see both the *Barstow* and the *Cauldrew*. They measure the angle between the line of sight to the *Barstow* and the line of sight to the *Cauldrew* as  $31^\circ$ . They radio the *Barstow* and by comparing known landmarks, find that the distance between the *Admiral* and the *Barstow* is 402 meters. The *Barstow* reports that an angle of  $70^\circ$  is found between their line of sight to the *Admiral* and their line of sight to the *Cauldrew*. To the nearest meter, what is the distance between the *Barstow* and the *Cauldrew*?

35. Given triangle  $ABC$  with  $b = 2$ ,  $c = 4$ , and  $m\angle A = 118^\circ$ , find  $a$ . Round the answer to two decimal places.

**Find the exact value of each variable.**

36.



37. A bag contains hair ribbons for a spirit rally. The bag contains 5 black ribbons and 7 green ribbons. Lila selects a ribbon at random, then Jessica selects a ribbon at random from the remaining ribbons. Find the probability that both events  $A$  and  $B$  occur. Express your answer as a fraction in simplest form.

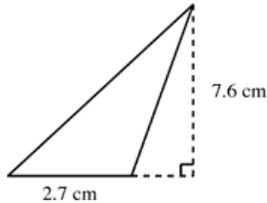
Event  $A$ : Lila selects a black ribbon.

Event  $B$ : Jessica selects a green ribbon.

38. Rewrite in exponential form and solve for  $x$ .  
 $\ln x = 2.39$  Two students tried this problem and got different results. Erica got 245.47, but Jorie got 10.91. Who solved it correctly? Which answer is correct?

**Find the area of the triangle.**

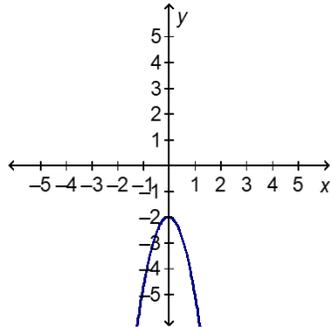
39.



40. What is the focus of the parabola given by the equation  $y = -3(x + 1)^2 - 5$ ?

41. What is the domain and range of  $y = \sqrt{x-2} - 2$ ?

42. What would happen to the graph if the value of  $a$  in  $y = ax^2 + bx + c$  changed from  $-3$  to  $3$ ?



43. Solve  $(x-5)^2 = 1$ .

44. Rewrite as a logarithm and solve for  $x$  to the nearest thousandth.  $5^x = 97$

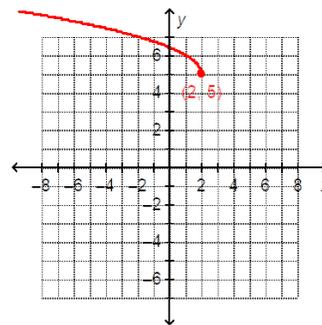
45. A bag contains hair ribbons for a spirit rally. The bag contains 5 black ribbons and 7 green ribbons. Lila selects a ribbon at random, then Jessica selects a ribbon at random from the remaining ribbons. Find the probability that both events  $A$  and  $B$  occur. Express your answer as a fraction in simplest form.  
 Event  $A$ : Lila selects a black ribbon.  
 Event  $B$ : Jessica selects a green ribbon.

46. Express the solution to  $5(e)^{24t} = 50$  as a logarithm.

47. Graph the solution to the inequality  $\sqrt{x+2} > 2$ ?

48. Graph the function  $f(x) = \frac{1}{2}x - 4$  for the domain  $x \geq -1$ . Classify the function as *discrete* or *continuous*. Then identify the range of the function.

49. Write the domain of the function graphed below in interval notation.



**Simplify the expression.**

50.  $(bc^3)^4$

**Find the sum or difference.**

51.  $(-7x^2 + 3) + (4x^2 + 2x - 1)$

**Find the product.**

52.  $(x-3)(x^2 + 4x + 5)$

**Factor the polynomial completely.**

53.  $x^3 - 2x^2 - 9x + 18$

**Simplify:**

54.  $3\sqrt{3} + 9\sqrt{3} - 4\sqrt{3}$

55. Is the value or  $\sqrt{9} + \sqrt{4}$  a rational or an irrational number.

56. Simplify the radical expression.

$\sqrt{\frac{27}{8}}$

57. Solve the equation, if possible.

$$\sqrt{2 - 3x} = -4$$

**Solve the linear system.**

58.  $2x + 5y = -2$

$$3x - 2y = 4$$

59. Are the functions  $f$  and  $g$  below inverses of each other?

$$g(x) = \frac{1}{3}x - \frac{1}{2}, f(x) = \frac{3}{2}(2x + 1)$$

60. The following equation describes the number of meters,  $x$ , which must be added to a string that measures 6 meters so that a pendulum will have a complete swing (back and forth) that lasts 6 seconds.

$$\sqrt{6 + x} = 3$$

How much longer should the string be so that the complete swing of the pendulum will be 6 seconds?

61. Growing up, you lived in a tiny country village.

When you left for college, the population was 840. You recently heard that the population has grown by 5%. What is the present population?

62. Solve for  $v$  in the equation  $t = \frac{u + v}{v}$ .

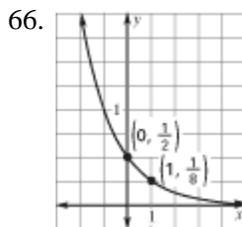
63. The cost of a 5 pound bag of dog food ranges from \$5.25 to \$5.95. Write and graph an inequality to describe this statement.

**Find the inverse of the function.**

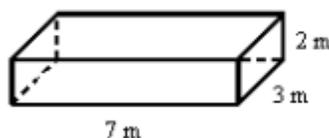
64.  $f(x) = \frac{3}{4}x^3 - 1$

65. The enrollment at Alpha-Beta School District has been declining 4% each year from 1994 to 2000. If the enrollment in 1994 was 1575, find the 2000 enrollment.

**Tell whether the graph represents exponential growth or exponential decay. Then write a rule for the function.**



67. Find the volume of the rectangular prism.



68. The volume of a cone is  $256\pi \text{ in}^3$  and the height is 12 inches. Find the radius.

69. A drawer contains 4 single red socks, 9 single white socks, and 2 single blue socks. Without looking, you draw out a sock, you replace it and draw out a second sock. What is the probability that the first sock and the second sock are both red?

70. Verify that  $f(x)$  and  $g(x)$  are inverses by showing that  $(f \circ g)x = x$  and  $(g \circ f)x = x$ .  $f(x) = 5x + 8$  and  $g(x) = \frac{1}{5}(x - 8)$ .

71. Graph the function  $f(x) = -x^2 + 2x + 8$ . Show intercepts, maxima, and minima.

72. Does the parabola open up or down?  $y = 4 + 6x - 2x^2$

73. Given:  $f(x) = \sqrt{x - 8}$  and  $g(x) = x^2 + 9$ , find  $(f \circ g)x$ .

**Find all real zeros of the function.**

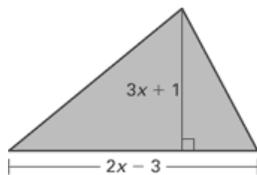
74.  $p(x) = 2x^3 - 42x + 40$

75. Describe the end behavior of the function

$$f(x) = 3x^3 + 4x^4 - 2x^2 - 5.$$

76. Simplify the expression  $\frac{x^2 + 4x}{x^2 - 16}$ .

77. Find an expression for the area of the figure.



Let  $f(x) = 1 - x^2$  and  $g(x) = \frac{1}{x}$ . Find the indicated value.

78.  $g(f(-5))$

79. Let  $f(x) = 6x$ . Find  $f^{-1}$ .

80. Are the functions  $f$  and  $g$  below inverses of each other?

$$g(x) = \frac{1}{3}x - \frac{1}{2}, \quad f(x) = \frac{3}{2}(2x + 1)$$

81. The Modern Grocery has cashews that sell for \$4.75 a pound and peanuts that sell for \$2.50 a pound. How much of each must Albert, the grocer, mix to get 90 pounds of mixture that he can sell for \$3.00 per pound. Express the problem as a system of linear equations and solve using the method of your choice to find the solution of the problem.

82. The projected worth (in millions of dollars) of a large company is modeled by the equation

$y = 256(1.04)^x$ . The variable  $x$  represents the number of years since 1997. What is the projected annual percent of growth, and what should the company be worth in 2007?

83. The volume of a cone is  $256\pi \text{ in}^3$  and the height is 12 inches. Find the radius.

84. A company has a spherical storage tank which is in need of painting. The radius of the tank is 35.4 ft. The type of paint used will cover approximately  $160 \text{ ft}^2$  per gallon. How many gallons of paint will be needed? (Round decimal to the higher whole number of gallons.)

85. A satellite is made of a cylinder and two hemispheres. The hemispheres have the same radius as the cylinder and each fit snugly on either end of the cylinder. If the diameter of the cylinder is 9 m and its length is 19 m, find the volume of the satellite. Leave your answer in terms of  $\pi$ .

86. A drawer contains 4 single red socks, 9 single white socks, and 2 single blue socks. Without looking, you draw out a sock, you replace it and draw out a second sock. What is the probability that the first sock and the second sock are both red?

87. A piece of fabric measures 39 inches by 42 inches. A triangular scarf with a height of 21 inches and a base of 22 inches is cut from the fabric. How much is left over?

88. In this exercise, refer to the division problem  $(4x^4 - 20x^3 + 23x^2 + 5x - 6) \div (x - 3)$ .

a. Find the quotient using long division.

b. Find the quotient using synthetic division.

c. Explain why you subtract in the process of long division, but add when using synthetic division.

d. Under what conditions can you use synthetic division to determine the quotient? Give an example where synthetic division would not be a good option.

e. What is the remainder for this problem? What information does this provide about  $(x - 3)$ ?

f. List the possible rational zeros of the equation  $f(x) = 4x^4 - 20x^3 + 23x^2 + 5x - 6$ .

**g. Critical Thinking** Use long division to divide  $(2x - 3)$  into  $4x^4 - x^2 - 2x + 1$ . Then use synthetic division. What do you notice about your solutions? Explain why it is still possible to use synthetic division if the leading coefficient of the linear expression is not 1. Clearly indicate why one must be careful when using this method.

89. By halftime of the football game, a total of 72 hot dogs and hamburgers have been sold for a total revenue of \$118.50. How many hamburgers have been sold? *Hint:* Set up and solve a system of equations to solve the problem.

Concession Sales	
Hot Dogs	\$1.25
Hamburgers	\$2.00

90. Compare the quiz grades of the two algebra classes shown in the table by comparing the measures of central tendency and variation of the two data sets.

First Period	10	5	6	5	6	7	8	5	6	2
Second Period	2	10	10	4	2	5	1	10	9	7

91. Describe how to obtain the graph of  $y = \sqrt{x+2} + 4$  from the graph of  $y = \sqrt{x}$ .

92. The function  $C = \frac{5}{9}(F - 32)$  converts degrees

Fahrenheit  $F$  to degrees Celsius  $C$ . The function  $K = C + 273$  converts degrees Celsius  $C$  to degrees Kelvin  $K$ . Find the composition of the function  $K$  with the function  $C$  and explain what it represents.

93. Here are the points scored by the Culver Eagles in their basketball games this year.

Home: 86, 81, 78, 94, 80, 69, 82, 49, 93, 87, 90  
 Away: 71, 65, 57, 84, 69, 67, 59, 72, 58, 64, 68

- Find the mean, median, range, and standard deviation for the points scored during the Eagles' home games this year.
- Find the mean, median, range, and standard deviation for the points scored during the Eagles' away games this year.
- Compare the statistics for home and away games. What can you conclude?
- Delete the 49 from the home games and recalculate the values in part (a). Do your conclusions from part (c) change? Explain.

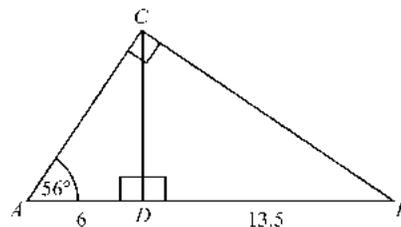
94. Explain the difference between a common logarithm and a natural logarithm.

95. A farmer wants to fence off a portion of a square field for a vegetable garden. The length of the garden will be 4 feet less than the length of the square field. The width of the garden will be 8 feet less than the length of the square field.

- Using  $x$  as the length of the square field, write an expression for the area of the garden.
- If the area of the garden will be 192 square feet, what are the dimensions of the vegetable garden?
- Write the function  $f(x) = (x - 4)(x - 8) - 192$  in standard form and explain how to use the graph of  $f(x)$  to check your answer in part (b).

96. Open-ended Problem: A plumber charges \$20 an hour. For each job he will charge an additional \$25 for the service call. Write an equation that represents the plumber's total charge with respect to the number of hours worked. Find the inverse of your equation. What does the inverse equation represent?

97. Drawing the altitude  $\overline{CD}$  from the right angle to the hypotenuse in right triangle  $ABC$  creates two new right triangles,  $\triangle ACD$  and  $\triangle CBD$ , which are similar to each other and also to  $\triangle ABC$ . (The figure may not be drawn to scale.)



**Part A** Find the measures of  $\angle ACD$ ,  $\angle BCD$ , and  $\angle CBD$ .

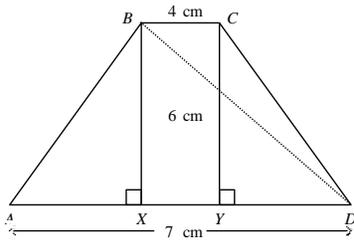
**Part B** Which angles and sides in  $\triangle ACD$  and  $\triangle CBD$  correspond to each other? Explain your answer.

**Part C** Write a proportion that can be solved to find  $CD$ . Solve it and show your work.

**Part D** What is the length  $AC$ ? Show the equation used and the steps used to solve it to find  $AC$  to the nearest tenth of a unit.

98. Use the equation  $y = \frac{x^2 - 4x - 21}{x + 3}$ .
- What is the domain of the equation? Explain.
  - Graph  $y = \frac{x^2 - 4x - 21}{x + 3}$ . Use an open circle for a point of discontinuity, a point where the function is undefined.
  - Compare your graph from part (b) with the graph of  $y = x - 7$ .

99. Use the law of cosines to investigate the lengths of the diagonals of the isosceles trapezoid with bases 4 centimeters and 7 centimeters long and a height of 6 centimeters. Find  $XY$ ,  $AX$ ,  $AB$ , and  $\cos A$ . Use this information to find  $BD$ .



100. A roofer says that he will only walk on a roof that has a vertical rise of 42 cm per 100 cm run. What angle does the roof make with the attic floor?
- Part A** Draw a diagram to model the situation.
- Part B** Write an appropriate trigonometric equation to solve the problem.
- Part C** Use technology to solve the equation to the nearest tenth.