

Pre-Calculus Summer Review Packet

DUE THE FIRST DAY OF SCHOOL

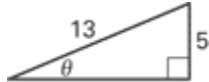
The problems in this packet are designed to help you review topics from previous mathematics courses that are essential to your success in Calculus or Statistics. **You are expected to bring this completed packet to class on the first day of school.** In addition, this packet will count as part of your first quarter grade. **Upon returning, you will be ASSESSED on the content of this packet.** All contents outlined in the packet are Pre-Calculus objectives. Neatly **SHOW YOUR WORK** on a separate sheet of paper.

***Remember, this will be part of your quarter one grade and 10 points will be deducted per day if this assignment is late.**

****Please note:** The decision to take Calculus or Statistics class is crucial. The class' assignments will require the following:

- ★ A desire to learn
- ★ Exemplary work habits
- ★ Exemplary time management skills
- ★ Personal responsibility for required assignments and attendance
- ★ Self-discipline
- ★ Determination to succeed

1. Simplify $\sin 86^\circ \cos 2^\circ + \cos 86^\circ \sin 2^\circ$?
2. What is the approximate value of θ in the triangle shown?



3. Which law is defined by $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$?
4. Find the value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$. Express your answer in degrees.
5. Given triangle ABC with $a = 7$, $C = 37^\circ$, and $B = 18^\circ$, find c . Round the answer to two decimal places.
6. Solve triangle ABC given that $A = 45^\circ$, $B = 54^\circ$, and $b = 70$.
7. Sean and Jackie made a shady area by stretching a sheet over a clothesline. The sheet was 2.5 m long and made an angle of 52° with the ground where it was anchored at each side. How wide was the shady area?
8. Given triangle ABC with $b = 2$, $c = 4$, and $m\angle A = 118^\circ$, find a . Round the answer to two decimal places.
9. Solve $8 \cos x + 4 = 0$ in the interval $0^\circ \leq x \leq 360^\circ$. The sum of the solutions is:
10. Suppose the depth of the tide in a certain harbor can be modeled by $y = 21 - 5 \cos\left(\frac{\pi}{6}t\right)$, where y is the water depth in feet and t is the time in hours. Consider a day in which $t = 0$ represents 12:00 midnight. For that day when are high and low tide and what are the depths of each?

11. Find $\sin(A + B)$ given that $\sin A = \frac{5}{7}$ with $\frac{\pi}{2} \leq A \leq \pi$ and $\cos B = \frac{5}{8}$ with $-\frac{\pi}{2} \leq B \leq 0$.
12. Solve $\tan(\pi + x) + 2 \sin(\pi + x) = 0$ for $0 \leq x \leq 2\pi$.

Simplify:

13. $\frac{7e^{19}}{35e^6}$
14. The formula $A = 2000e^{rt}$ can be used to find the dollar value of an investment of \$2000 after t years when the interest is compounded continuously at a rate of r percent. Find the value of the investment after 6 years if the interest rate is 7%. Find the value of the investment after 12 years if the interest rate is 8%.

Evaluate:

15. $\log_3 729$
16. Use the formula $R = \log_{10} I$, where R is the measurement of the Richter scale and I is the intensity, to find the Richter scale measurement of an earthquake with intensity 53,000,000.
17. Express as a single logarithm: $\log_2 13 + \log_2 60$

Solve:

18. $\frac{1}{9} = 27^{7x-6}$

Solve the equation. Check for extraneous solutions.

19. $\log_5(3x + 9) = 2$
20. If there are initially 2000 bacteria in a culture, and the number of bacteria double each hour, the number of bacteria after t hours can be found using the formula $N = 2000(2^t)$. How long will it take the culture to grow to 60,000 bacteria?
21. Write an equation for the geometric sequence: 3, 6, 12, 24, 48, ...?
22. Which vector has the greatest magnitude?
- | | |
|-----------------------------|-----------------------------|
| a. $\langle -3, -5 \rangle$ | c. $\langle 7, -8 \rangle$ |
| b. $\langle 2, 6 \rangle$ | d. $\langle -8, -3 \rangle$ |
23. Write the standard form of the equation of the circle with radius 7 and center at (0, 0).
24. A ship approaching shore is located by radio signals along a curve described by the equation $r = \frac{8}{2 + 2 \sin \theta}$. Which conic is represented by the equation

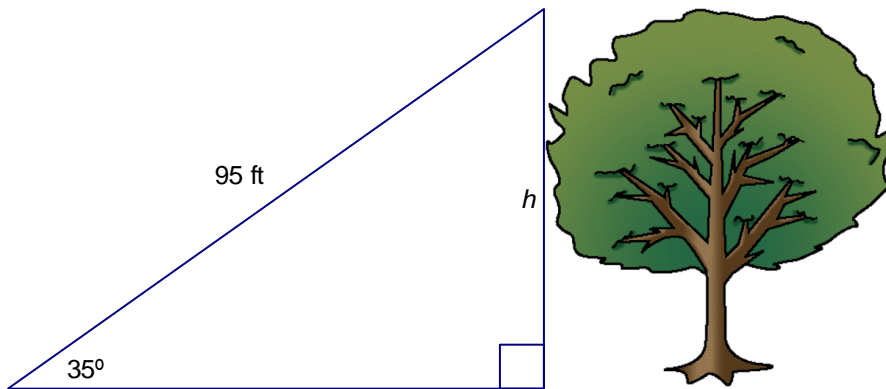
25. What is the center of the circle represented by the equation $(x - 9)^2 + y^2 = 25$
26. What is the radius of the circle represented by the equation $(x - 9)^2 + y^2 = 25$
27. Put the equation, $x^2 + y^2 - 10x + 5 = 0$, into graphing form.
28. Put the equation, $x^2 + y^2 + 4x - 8y + 16 = 0$, into graphing form.
29. Find the center and radius of the circle whose equation is $x^2 + y^2 + 12x - 10y + 45 = 0$.
30. Write the equation of the circle whose center is $(-8, 4)$ and has a radius of $5\sqrt{3}$.
31. What is the center of the ellipse given by the equation $\frac{(x-3)^2}{4} + \frac{y^2}{9} = 1$?
32. What type of major axis does the ellipse given by the equation $\frac{(x-3)^2}{4} + \frac{y^2}{9} = 1$ have?
33. What are the foci of the ellipse given by the equation $\frac{x^2}{16} + \frac{(y+1)^2}{4} = 1$?
34. Which equation represents an ellipse centered at $(1, -4)$ with a horizontal axis 8 units long and a vertical axis 6 units long?
35. Rewrite the equation $4x^2 + 9y^2 - 54y + 45 = 0$ in graphing form.
36. Rewrite the equation $x^2 + 4y^2 - 4 = 0$ in graphing form.
37. Write the graphing form of the equation of the ellipse with center at $(-1, 3)$, vertex at $(3, 3)$, and minor axis of length 2.
38. Find an equation of the ellipse with foci at $(-2, 3)$ and $(2, 3)$, and major axis of length 8.
39. What is the center of the hyperbola, $\frac{(x-1)^2}{16} - \frac{(y+3)^2}{4} = 1$?

40. What is the center of the hyperbola, $\frac{(y+2)^2}{9} - \frac{(x+3)^2}{9} = 1$?
41. Where are the foci of the hyperbola, $\frac{(y+2)^2}{9} - \frac{(x+3)^2}{9} = 1$?
42. Which hyperbola has a vertical transverse axis?
43. Rewrite the equation $9x^2 - 16y^2 + 18x + 153 = 0$ in graphing form.
44. Write the equation for the hyperbola in general form with vertices at (2, 3) and (2, -1) and foci at (2, 6) and (2, -4).
45. Find the center of the hyperbola: $16x^2 - 9y^2 + 160x - 54y + 175 = 0$
46. What is the vertex of the parabola given by the equation $y = -3(x+1)^2 - 5$?
47. What is the focus of the parabola given by the equation $y = -3(x+1)^2 - 5$?
48. What direction does the parabola given by the equation $x = -3x^2 + 3$ open towards?
49. What is the vertex of the parabola given by the equation $x = -3x^2 + 4$?
50. What is the equation of the directrix of the parabola given by the equation $x = 2(x-5)^2 + 1$?
51. What is the equation of the axis of symmetry of the parabola given by the equation $y = -2x^2 + 4$?
52. Write the equation of the parabola given the vertex (0, 4) and directrix $y = 2$.
53. Find the vertex of the parabola: $y^2 + 10y - 20x + 37 = 0$
54. Find the vertex of the parabola: $y^2 - 4y - 4x + 16 = 0$.
55. What is the vertex of the parabola given by the equation $y = -3(x+1)^2 - 5$?

56. What is the focus of the parabola given by the equation $y = -3(x + 1)^2 - 5$?
57. What direction does the parabola given by the equation $x = -3x^2 + 3$ open towards?
58. What is the vertex of the parabola given by the equation $x = -3x^2 + 4$?
59. What is the equation of the directrix of the parabola given by the equation $x = 2(x - 5)^2 + 1$?
60. What is the equation of the axis of symmetry of the parabola given by the equation $y = -2x^2 + 4$?
61. What is the magnitude of $\mathbf{v} = \langle 4, -9 \rangle$? Round to the nearest tenth if necessary.
62. Rod is flying his remote controlled airplane in a competition. The component vector $\langle -26, 21.9 \rangle$ represents the velocity and heading of the airplane. To the nearest mile per hour, how fast is the airplane traveling?

Verify the identity.

63.
$$\frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$
64. A satellite dish has a parabolic cross section with a focus that is 3 feet from the vertex. The cross section is placed on a coordinate plane with the vertex at $(0, 0)$ and opening to the right.
- Find the coordinates of the focus and the equation of the directrix. Explain your answers.
 - Write an equation for the cross section of the satellite dish. Explain your answers.
 - If the satellite dish is 4 feet deep, find the diameter of the satellite dish at its opening.
 - If the opening of the satellite dish has a circumference of 16π , how deep is the dish?
65. Write the expression that best describes the sequence: 224, 112, 56, 28,
66. Red maple trees can reach heights of up to 80 feet. What is the height of the maple tree shown below? Round to the nearest tenth if necessary.



67. Use a table and graph to solve $3^{2x} = 6561$.

68. \vec{AB} has its initial point at $(-5, -6)$ and its terminal point at $(8, -10)$. What is the component form of the vector?

69. An airplane is flying 160 miles per hour at a heading of 110° . The direction of the heading is with respect to due north, or the positive y -axis on a coordinate grid. How can you represent the velocity and direction of the plane as a component vector? Round to the nearest tenth if necessary.

70. A fruit basket contains 6 apples and 8 oranges. Sarah randomly selects one, puts it back, and then randomly selects another. What is the probability that both selections were oranges?

Write an equation for the conic section.

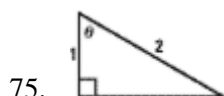
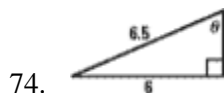
71. Ellipse with vertices at $(4, -7)$ and $(9, -7)$, and foci at $(0, -7)$ and $(8, -7)$

72. You deposit \$4000 in an account that pays 7% annual interest compounded continuously. Find the balance at the end of 5 years.

Solve the equation. Check for extraneous solutions.

73. $\log_2(x + 6) - \log_2(x - 1) = 3$

Evaluate the six trigonometric functions of θ .



76. Solve $\triangle ABC$ with $A = 33^\circ$, $a = 6.5$, and $b = 3$.

77. Solve triangle ABC given that $a = 19$, $b = 10$, and $c = 14$.
78. Solve $9 \cos x + 3 = 0$ in the interval $0^\circ \leq x \leq 360^\circ$.
79. Solve $2 \sin^2 x + 3 \sin x - 4 = 0$ in the interval $0 \leq x < \pi$.
80. Find $\sin(A + B)$ given that $\sin A = \frac{6}{7}$ with $\frac{\pi}{2} \leq A \leq \pi$ and $\cos B = \frac{2}{5}$ with $-\frac{\pi}{2} \leq B \leq 0$.

81. Condense the expression. $\frac{1}{5} \log_3 32 - 2 \log_3 x + \frac{1}{2} \log_3 y$

82. Solve. $6^{-0.2x} - 3 = 7$

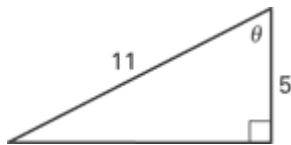
Solve the equation. Check for extraneous solutions.

83. $\ln(x + 7) = \ln(3x - 5)$

84. Marion decides to invest \$6000 at 5% interest compounded continuously. Find the value of the investment after seven years.

85. A ladder 12 feet long makes an angle of 67° with the ground as it leans against a store. How far up the side of the store does the ladder reach?

86. Find the measure of the angle θ for the triangle shown below in radians and degrees.



87. A hunter at the top of a 75-foot cliff looks down upon a grassy plain below and spots an elk. The angle of depression to the elk is 25° . How far is the elk from the base of the cliff?
88. A boat travels 50 miles due west before adjusting its course 25° north of west and traveling an additional 35 miles. How far is the boat from its point of departure?
89. Calculate the probability of randomly guessing at least 7 correct answers on a 10-question true-or-false quiz to get a passing grade.
90. You are flying a kite and want to know its angle of elevation. The string on the kite is 39 meters long and the kite is level with the top of a building that you know is 24 meters high. Use an inverse trigonometric function to find the angle of elevation of the kite.
91. Identify the focus and directrix of the parabola given by $x^2 = 12y$.

92. Mary is in a canoe crossing a river that has a current flowing at 3.2 meters per second due east. Mary begins paddling across the river at 3.4 meters per second at an angle of 70° with the bank. Find the magnitude and direction of the canoe's path.
93. Find the vertical component of \mathbf{v} with a magnitude of 6.1 inches and a direction angle of 55° .

Solve the equation.

94. $7(7 - 5x) = 3(4 + x)$

95. $|x - 4| = 15$

96. $8^{(x+3)} = \left(\frac{1}{4}\right)^{3x-6}$

Solve the inequality. Graph the solution on a number line.

97. $-2x + 8 > 3x + 10$

Evaluate the function for the given value of x .

98. $f(x) = 3|x - 4| + 2; f(2)$

99. Write an equation of the line that passes through $(5, -2)$ and is parallel to the line $2x - 3y = 6$.

Graph the system of linear inequalities.

100. $2x + y \geq 1$
 $x \leq 3$

Use Cramer's rule to solve the linear system.

101. $5x - 2y + 7z = 12$
 $2x + 5y + 3z = 10$
 $3x - y + 4z = 8$

102. You have \$18 to spend for lunch during a 5 day work week. It costs you about \$1.50 to make a lunch at home and about \$5 to buy a lunch. How many times each work week should you make a lunch at home?
103. Write $y = 3x^2 - 108$ in intercept form and give the function's zeros.
104. Use the quadratic formula to solve $6x^2 + 2 = 7x$.

Graph the quadratic inequality.

105. $y < 2x^2 + 12x + 15$

Write a quadratic function whose graph has the given characteristics.

106. vertex: $(-3, 2)$
point on graph: $(-1, -18)$

Simplify the expression. Tell which properties of exponents you used.

107. $\frac{x^9}{x^{-2}}$

Describe the end behavior of the graph of the polynomial function. Then evaluate the function for $x = -4, -3, -2, \dots, 4$. Then graph the function.

108. $y = (x - 3)(x + 1)(x + 2)$

Perform the indicated operation.

109. $(x - 4)(x + 1)(x + 3)$

Factor the polynomial.

110. $2x^3 - 3x^2 + 4x - 6$

List all the possible rational zeros of f using the rational zero theorem. Then find all the zeros of the function.

111. $f(x) = x^3 - 5x^2 - 14x$

112. An adult human body contains about 75,000,000,000,000 cells. Each is about 0.001 inch wide. If the cells were laid end to end to form a chain, about how long would the chain be in miles? Give your answer in scientific notation.

Simplify the expression. Assume all variables are positive.

113. $\left(\frac{81x^2}{y}\right)^{3/4}$

Find the inverse function.

114. $f(x) = \frac{1}{3}x - 4$

Graph the function. Then state the domain and range.

115. $f(x) = -2x^{1/2} + 4$

Solve the equation. Check for extraneous solutions.

116. $\sqrt[3]{7x-9} + 11 = 14$

117. $\log_6 x + \log_6 (x+5) = 2$

118. Some biologists study the structure of animals. By studying a series of antelopes, biologists have found that the length l (in millimeters) of an antelope's bone can be modeled by $l = 24.1d^{2.8}$ where d is the midshaft diameter of the bone (in millimeters). If the bone of an antelope has a midshaft diameter of 20 millimeters, what is the length of the bone?

Graph the function. State the domain and range.

119.

$$y = 3\left(\frac{1}{3}\right)^x + 2$$

120.

$$y = \log_3(x-2)$$

The variables x and y vary inversely. Use the given values to write an equation relating x and y . Then find y when $x = 3$.

121. $x = 12, y = \frac{2}{3}$

Graph the function.

122. $y = \frac{x-5}{x+2}$

Perform the indicated operation. Simplify the result.

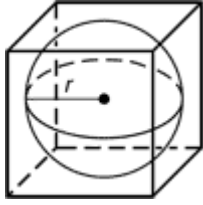
123. $\frac{7x}{2x+3} - \frac{9x}{x-3}$

124. $\frac{x^3+x^2}{x^2-16} \cdot \frac{x+4}{3x^4+x^3-2x^2}$

Solve the equation using any method. Check each solution.

125. $\frac{5}{x+3} - \frac{3}{x-2} = \frac{5}{3x-6}$

126. A sphere with radius r is inscribed in a cube as shown. Find the ratio of the volume of the cube to the volume of the sphere. Write your answer in simplified form.



Find the distance between the two points. Then find the midpoint of the line segment connecting the two points.

127. $(-1, 6), (2, 8)$

Graph the equation.

128. $x^2 + y^2 = 36$

Classify the conic section and write its equation in standard form.

129. $-x^2 + y^2 - 6x - 6y - 4 = 0$

Find the points of intersection, if any, of the graphs in the system.

130. $y^2 - 8x^2 = 9$
 $y = 4x^2 - 3$

131. Find the number of distinguishable permutations of the letters in FLORIDA.

Expand the power of the binomial.

132. $(3x - y)^3$

133. When two six-sided number cubes are tossed, there are 36 possible outcomes. Find the probability that the sum is less than or equal to 3.

Students are taking a science quiz that consists of 8 multiple choice questions. Each question has 5 possible responses. One student, who forgot to study, guesses each answer at random from the five choices. What is the probability the student correctly answers each number of questions?

134. at least 4

135. Write a set of seven numbers for which the mean is 5, the mode is 7, and the median is 6.

Write the next term of the sequence, and then write a rule for the n th term.

136. 2, 4, 8, 16, ...

Write the first six terms of the sequence.

137. $a_1 = -8$
 $a_n = a + 6$

Find the sum of the series.

138. $\sum_{i=1}^{100} i$

139. Find the sum of the infinite geometric series $2 + 1 + 0.5 + 0.25 + \dots$.

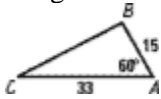
140. An object is dropped from an airplane. During the first second, the object falls 4.9 meters. During the second second, it falls 14.7 meters. During the third second, it falls 24.5 meters. During the fourth second, it falls 34.3 meters. If this pattern continues, how far will the object fall during the tenth second? Find the total distance the object will fall after 10 seconds.

Evaluate the function without using a calculator.

141. $\tan\left(-\frac{\pi}{4}\right)$

Solve $\triangle ABC$.

142. Image:



Draw one cycle of the function's graph.

143. $y = -3 + 2 \cos(x - \pi)$

Simplify the expression.

144. $\frac{\tan 2x}{2 \tan x} - \frac{\sec^2 x}{1 - \tan^2 x}$

Verify the identity.

145. $\cos 3x = \cos^3 x - 3 \sin^2 x \cos x$

Find the exact value of the expression.

146. $\cos 375^\circ$

147. The average daily temperature T (in degrees Fahrenheit) in Baltimore, Maryland, is given in the table. The variable t is measured in months, with $t = 0$ representing January 1. Use a graphing calculator to write a trigonometric model for T as a function of t .

Source: U.S. National Oceanic and Atmospheric Administration

t	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5	10.5	11.5
T	75	79	87	94	98	101	104	105	100	92	87	77

148. Graph the function $y = 2(3)^{-2x}$ on a graphing calculator. As x increases what happens to the y -values of the function?
149. Graph $y = e^{2x - 1}$ and $y = 20$ on a graphing calculator. Find the solution to $e^{2x - 1} = 20$ by finding the intersection of the two graphs.
150. Evaluate $300e^{-0.076t}$ when $t = 15$.

